

Enrollment No: _____

Exam Seat No: _____

C.U.SHAH UNIVERSITY

Summer Examination-2016

Subject Name: Mathematics-II

Subject Code: 4SC02MTC1

Branch: B.Sc.(All)

Semester: 2 Date: 13/05/2016

Time: 10:30 To 1:30

Marks: 70

Instructions:

- (1) Use of Programmable calculator & any other electronic instrument is prohibited.
- (2) Instructions written on main answer book are strictly to be obeyed.
- (3) Draw neat diagrams and figures (if necessary) at right places.
- (4) Assume suitable data if needed.

- Q-1** **Attempt the following questions:** **(14)**
- a) State De'Moivre's theorem. **(02)**
 - b) Prove that $\cosh^2 x - \sinh^2 x = 1$. **(02)**
 - c) Evaluate: $\int_0^{\frac{\pi}{2}} \cos^6 x \, dx$. **(02)**
 - d) Solve: $(D^2 + 6D + 9)y = 0$. **(02)**
 - e) $\frac{1}{f(D)}(e^{mx} x^2) = e^{mx} \frac{1}{f(D+m)} x^2$. Determine whether the statement is true or false? **(01)**
 - f) Define: Oscillatory Sequence. **(01)**
 - g) Find polar form of $1 - i\sqrt{3}$. **(01)**
 - h) Write a condition that the general equation of second degree represents a cone. **(01)**
 - i) Any line on the surface of a cylinder is called its _____. **(01)**
 - j) $\frac{1}{D-m} X = e^{-mx} \int X e^{mx} \, dx$. Determine whether the statement is true or false? **(01)**

Attempt any four questions from Q-2 to Q-8

- Q-2** **Attempt all questions** **(14)**
- a) Solve: $x^6 - 1 = 0$. **(05)**
 - b) Expand $\frac{\sin 6\theta}{\sin \theta}$ in terms of increasing power of $\cos \theta$. **(05)**
 - c) Prove that $\tan^{-1} \left(i \log \frac{a-ib}{a+ib} \right) = \frac{2ab}{a^2-b^2}$. **(04)**
- Q-3** **Attempt all questions** **(14)**
- a) Find equation of cylinder whose generators intersect the conic $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0; z = 0$. **(05)**
 - b) Prove that $\cos^8 \theta = \frac{1}{128} [\cos 8\theta + 8\cos 6\theta + 28\cos 4\theta + 56\cos 2\theta + 35]$. **(05)**
 - c) Prove that $\sinh^{-1}(\tan x) = \log \tan \left(\frac{\pi}{4} + \frac{x}{2} \right)$. **(04)**



- Q-4** **Attempt all questions** (14)
- a) Find equation of cone whose vertex is the point (1, 1, 1) and whose guiding curve is $x^2 + z^2 = 4, y = 0$. (05)
- b) Prove that $\lim_{n \rightarrow \infty} \sqrt[n]{n} = 1$. (05)
- c) Find the equation of the enveloping cylinder of the sphere $x^2 + y^2 + z^2 = 25$, whose generators are parallel to the line $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$. (04)
- Q-5** **Attempt all questions** (14)
- a) Check whether the equation $x^2 - 2y^2 + 3z^2 - 4xy + 5yz - 6xz + 8x - 19y - 2z - 20 = 0$ represent a cone with vertex (1, -2, 3). (05)
- b) Prove that equation of right circular cylinder whose axis is the line $\frac{x-\alpha}{l} = \frac{y-\beta}{m} = \frac{z-\gamma}{n}$ and radius r is $(x - \alpha)^2 + (y - \beta)^2 + (z - \gamma)^2 - \frac{[l(x-\alpha)+m(y-\beta)+n(z-\gamma)]^2}{l^2+m^2+n^2} = r^2$ (05)
- c) Prove that $\lim_{n \rightarrow \infty} \frac{1+\frac{1}{3}+\frac{1}{5}+\dots+\frac{1}{2n-1}}{n} = 0$. (04)
- Q-6** **Attempt all questions** (14)
- a) Prove that $\int \cos^n x \, dx = \frac{\cos^{n-1} x \sin x}{n} + \frac{n-1}{n} \int \cos^{n-2} x \, dx$. (05)
- b) Solve: $y'' - 5y' + 6y = 0; y(1) = e^2, y'(1) = 3e^2$. (05)
- c) Evaluate: i) $\int_0^{\frac{\pi}{2}} \sin^6 x \cos^8 x \, dx$, ii) $\int_0^{\frac{\pi}{2}} \sin^4 x \cos^5 x \, dx$. (04)
- Q-7** **Attempt all questions** (14)
- a) Solve: $3 \frac{d^2y}{dx^2} + 2 \frac{dy}{dx} - y = e^{\frac{x}{2}} + 2e^{3x}$. (05)
- b) Solve: $(D^3 + 3D^2 + 2D)y = x^2$. (05)
- c) Solve: $(D^3 + 1)y = 0$. (04)
- Q-8** **Attempt all questions** (14)
- a) Solve: $(x^2 D^2 - 3xD + 4)y = x^2$. (05)
- b) Solve: $\frac{dx}{dt} = 5x + y, \frac{dy}{dt} = y - 4x$. (05)
- c) Identify the given surface $9x^2 + 4y^2 - 9z^2 - 18x - 8y - 18z = 32$. (04)

